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it. I cannot see any real difficulty here. Our faith in simplicity, so far as science depends on it, is by no means a belief that the simple is as such more probable than the complex; we should in general even incline to the contrary opinion. Our faith merely declares that an unmotived complexity is dangerous; or, more explicitly, that, other things being equal, the errors arising from a simple hypothesis will probably be less than those arising from a complex one; for even should the truth itself be exceedingly complex, the simple hypothesis is likely to be closer to it than the more complex one. But this is not paradox; it is common sense.

A chapter upon the "Calculus of Probabilities" is seriously weakened by an indefensible distinction between subjective and objective probability, the latter conception being a mere mass of direct self-contradictions. If one observes a game of chance "*a long time,*" we are told, "he *will* find that events *have taken place* in conformity with the laws of the calculus of probabilities" (p. 132); and this is the phenomenon of objective probability which M. Poincaré confesses himself helpless to explain. But the phenomenon is simply a confusion of two points of view, the uncertainty before and the certainty after the event. The conformity to the calculus, if it occurs, is a fact and not a probability — a fact, moreover, with which science is quite competent to deal.

I fear that the reader has been given but a slight notion of the exceeding interest and suggestiveness of this work. If there is much that should awaken caution, there is also a rich fund of wise and penetrating observations. Those who are least attracted by the author's conclusions may well be repaid for the reading by the impressive survey which he gives of the present state of mathematical and physical science.

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Erkenntnis und Irrtum: Skizzen zur Psychologie der Forschung. Von E. MACH. Leipzig, J. A. Barth, 1905. — pp. ix, 461.

Science is part of the human animal's reaction to its surroundings. If so, it must demonstrably serve the common end of such reactions — that of self-preservation. But the ease with which a biological function is ascribable to this or that detail of behavior varies from reaction to reaction. Obviously, we may more readily establish the service rendered by eating to the general end of living, than detect in a work on differential equations the perfecting of an arm with which to carry on the struggle for existence. To give to these higher products of our knowing a place among our 'reactions to environment' by catching the teleological aspect of each is, then, a task not only worth doing for the sake of a certain continuity thereby won for our thought, but, one suspects, difficult of accomplishment.

The idea is not of course new. As a matter of personal reminiscence, the reviewer would cite two little works of Schneider, *Der thierische Wille* and *Der menschliche Wille*, which attracted favorable notice in their day. But the problem has longer ancestry and larger relationship in the history of thought. The interest of seeking a function for bodily parts apparently contributing nothing to the purposes of life is as old as physiology; while the idea of bringing out the teleological aspect of behavior not associated with a special organ is beautifully illustrated in Darwin's *Expression of the Emotions*. Development of the problem consists, I take it, in the extension of the concept of 'behavior' to regions not contemplated by those who have previously interested themselves in the research of 'functions.' Hence the novelty of the work before us is to be judged by the generality of its ideal: "Der Naturforscher kann zufrieden sein, wenn er die bewusste, psychische Tätigkeit des Forschers als eine methodisch geklärte, verschärfte und verfeinerte Abart der instinctiven Tätigkeit der Tiere und Menschen wiedererkennt, die im Natur- und Kulturleben täglich geübt wird" (p. iv).

To contribute to the realization of such an ideal no one could be better equipped than Mach, with his unsurpassed command of the history of science. As some measure of his reach, we may take an example or two from his later chapters, "Deduktion und Induktion in psychologischer Bedeutung," "Zahl und Mass," "Der physiologische Raum im Gegensatz zum metrischen," "Zur Psychologie und natürlichen Entwickelung der Geometrie," "Die physiologische Zeit im Gegensatz zur metrischen." These sketches deal with the biological aspects of the more mathematical sciences; the problems of mechanics and physics, together with certain technical terms connected with the methods of those sciences (such as 'analogy,' 'hypothesis,' 'problem') having served as topics to earlier chapters.

In estimating the function of logical deduction, Mach has not advanced beyond the thought of Mill: deduction is not an instrument for the discovery of new truths; in the end it yields no more in the way of 'Erfahrung' than was furnished it at the outset. Its function is to serve as a labor-saving device. "Dieselbe (logische Operation) bringt uns die Abhängigkeit der Erkenntnisse von einander zu klaren Bewusstsein und erspart uns eine besondere Begründung für einen Satz zu suchen, der schon in einem andern enthalten ist" (p. 302).

If we are merely looking for some function that can be assigned to logic, no doubt one is here indicated. But if we ask of a discussion of logic that it contribute something to the settlement of historical issues, we are tempted at this point to lament lost opportunities. On the one hand, the 'psychology of logic' has never been more thoroughly threshed over than in current literature. This material Mach has simply ignored, and his results would be judged ultra-conservative by those most in sympathy with his motives. On the other hand, the anti-psychological tendencies of the older rationalism are dismissed, we may assume, with the observation that logic is no 'instrument of discovery.' Yet surely the psychology of rationalism would offer a beautiful illustration of the part that 'economy of thought' has played in inspiring human effort. And the outcome is somewhat more than a horrible example.

The treatment of number, the attempt "den Ursprung der Zahlvorstellung und des Zahlbegriffes aus dem unmittelbaren oder mittelbaren *biologischen* Bedürfnis psychologisch aufzuklären" (p. 318), is too detailed to admit of more than a limited reproduction. The needs of individual life demand a simple number concept; social intercourse, particularly trade, leads to the invention of a more or less economical notation for larger numbers; the operations of arithmetic are so many short cuts of counting; the later algebraic formulation of the laws of arithmetic facilitates and extends its application. The history of the development of our complex number system and its arithmetic, the construction of a number continuum, are easily shown to have an economical motive, as soon as we understand the more technical services they render the scientist.

All this is set forth with a wealth of illustration that makes the sketch instructive, paragraph by paragraph. But it may also seem in conclusion somewhat obvious unless it is saved by a certain polemical intention. This intention is indicated in the following remark: "Man bezeichnet die Zahlen oft als 'freie Schöpfungen des menschlichen Geistes.' . . . Das Verständnis dieser Schöpfungen wird aber weitmehr gefördert wenn man den *instinctiven Anfängen* derselben nachgeht und die Umstände betrachtet welche das Bedürfnis nach diesen Schöpfungen erzeugten. Vielleicht kommt man dann zur Einsicht dass die ersten hierher gehörigen Bildungen unbewusst und biologisch durch materielle Umstände *erzwungen* waren . . ." (p. 322).

Whose is the view with which Mach is here contrasting his own? The name of Kant is not brought into this (as it is into many another) discussion; yet if the author is not in his own mind opposing his results to an *a priori* theory of number, one is at a loss to place the historical reference.

Will the reviewer be judged captious, if, after complaining that the treatment of logic was too feebly historical in its reference, he now finds that the study of number is not justified in pronouncing on such an historical issue? And yet would not Kant, who insisted that experience is our only teacher, be astonished to find his doctrine of *a priori* number opposed on the ground that the progress of our learning from experience could be classed among the phenomena of an organism's adaptation to its environment?

On the other hand, a real issue seems to lurk somewhere in the implied attitudes of the Kantian and of the Empiricist. For Kant, number was a concept which no possible experience could fail to illustrate, seeing that experience involved manifoldness in its meaning. For Mach, "würde die physische *Erfahrung* nicht lehren *dass eine Vielheit aequivalenter, unveränderlicher, beständiger Dinge existiert* . . . so hätte das Zählen gar keinen Zweck und Sinn" (p. 324). We have only to accept the implication that 'Erfahrung' *might*, without thereby ceasing to be 'Erfahrung,' fail to reveal the existence of this manifold, just as it might fail to present Jupiter with four satellites, and the issue between the two schools is drawn. Only, to the deciding of this issue, the method to which Mach gives such elegant expression can contribute nothing. It surely throws no light on the question as to whether experience could exist otherwise than as a numerable manifold to point out, say, that an accident of human anatomy conducted us to a decimal system of notation; that, had we been a twelve-fingered race, we should probably have developed a duodecimal system, to which, as it is, other considerations of utility may yet conduct us. And the like of other analyses of the empirical needs that are reflected in our developed number system and its arithmetic. In a word, the *limits* to the variability of experience cannot be treated by any method which confines itself to showing that *if* the condition *A* varied, the result *B* would vary with it.

I have come perhaps too quickly to this estimate of what our author's method can accomplish, and in what respects it promises to throw no new light on historical problems. I am reduced to repeating much the same comment on the chapters that deal with geometry and chronometry. In both cases, to be sure, we have problems worthy of discussion for their own sake. Thus, confining ourselves to the first topic, we are aware that perceptual space and the space of geometry are different enough; indeed, when we come to compare the two we are reduced to such points of correspondence as those which Mach enumerates: "Beide Räume sind dreifache Mannigfaltigkeiten. Jeden Punkt des geometrischen Raumes A, B, C, D . . . entspricht ein Punkt A′ B′ C′ D′ . . . des physiologischen Raumes. Wenn C zwischen B und D liegt, so liegt C′ zwischen B′ und D′. Man kann auch sagen: einer kontinuirlichen Bewegung im geometrischen Raum entspricht eine kontinuirliche Bewegung des zugeordeten Punktes im physiologischen Raum." The extent of this divergence of the two spaces introduces the question: "Wie kommt es nun, dass der physiologische Raum vom geometrischen sosehr verschieden ist? Wie gelangt man doch von erstern Vorstellungen allmählich zu den letzteren" (p. 337 f.)

Every elementary sensation consists of two factors: quality and local sign (*Sinnesempfindung* and *Organempfindung*). "Man kann sagen, dass der physiologische Raum ein System von *abgestuften Organempfindungen* ist." This arrangement of the discrete Organempfindungen in a coördinated system is the outcome of the organism's repeated reactions to stimulation . . . "Die vollkommenste gegenseitige biologische Anpassung einer Vielheit von Elementarorganen kommt eben in der räumlichen Wahrnehmung besonders deutlich zum Ausdruck" (p. 339).

It is the experiences which follow on locomotion that conduct us, by an imperceptible transition, from this anisotropic and limited space perception to the geometer's space concept. "Die beliebige Locomotion des Leibes als Ganzes und die Möglichkeit beliebiger Orientierung fördern die Einsicht dass wir überall und nach allen Richtungen dieselbe Bewegungen ausführen können, dass der Raum überall und nach allen Richtungen *gleich* beschaffen, und dass derselbe als unbegrenzt und unendlich vorgestellt werden kann" (p. 341).

This is an excellent portrayal of the experiences that guide us in the conduct of life to the construction, first of a physiological, then of a geometrical space. The progressive utility to the organism of these successive constructions is made obvious, and as the fixing of this utility is more difficult than in the case of the development of the number concept, the study has so much the more value for the psychologist. But here again the author does not feel that the interest of his effort is exhausted in having brought clearly to light the biological importance of the space concept and of its geometry. He feels that the outcome gives him a right to pronounce on certain historical issues, and now it is explicitly the Kantian doctrine that is placed in contrast. "Kant hat behauptet: 'Man kann sich niemals

eine Vorstellung davon machen, dass kein Raum sei, ob man sich gleich ganz wohl denken kann, dass keine Gegenstände darin angetroffen werden.' Heute zweifelt kaum jemand daran, dass die Sinnesempfindungen und die Raumempfindungen nur *miteinander* ins Bewusstsein treten und aus demselben wieder verschwinden können. Dasselbe muss wohl von den betreffenden Vorstellungen gelten. Wenn für Kant der Raum kein 'Begriff' sondern eine 'reine [blosse?] Anschauung *a priori*' ist, so sind die heutigen Forscher sehr geneigt, den *geometrischen* Raum für einen *Begriff*, und zwar für einem durch *Erfahrung* erworbenen Begriff, zu halten" (p. 344).

Here, again, the student of the history of philosophy must, if I am not mistaken, have the impression that the author's method does not justify him in comparing his results with Kant's. Not but that he is warranted in calling the space concept used in geometry a 'Begriff', and insisting that the motives that lead us to the formation of such a concept are to be sought in the character of our experiences. What, then, one asks, is to prevent Mach from opposing 'Begriff' to 'Anschauung,' and further an 'Erfahrungsbegriff' to a 'reine Anschauung,' since Kant invites just this manner of contrasting portions of our knowledge?

I answer with enforced brevity that Kant and Mach are using different tongues. Kant meant, in distinguishing 'Anschauung' from 'Begriff,' to contrast two kinds of relationship whose 'formal properties' were as different as those, say, distinguishing 'between' from 'like.' Mach means by a 'Begriff' any notion that involves 'abstraction' from content (*Anschauung*). "Das blosse System der Raumempfindungen können wir nicht anschauen; wir können aber von den als nebensächlich betrachteten Sinnesempfindungen *absehen*, und wenn man diesen leicht und unvermerkt vor sich gehenden Prozess nicht genug beachtet, kann leicht der Gedanke enstehen, man habe eine reine Anschauung vollzogen" (p. 344). Kant meant the phrase, "unabhängig von Erfahrungen," to denote any concept which is necessary to define what the term 'Erfahrung' means. Mach means by a 'durch Erfahrung erworbener Begriff' one that is developed in the individual and in the race in response to stimulation. Kant's 'Anschauungen *a priori*,' then, were no less abstractions from experience in this sense than are Mach's. But Kant had the ingenious idea of asking: Are there no limits to the variations we can suppose possible in experience? And he answered that if 'experience' has a meaning, all that is involved in this meaning is indispensable to experience. Individuality and manifoldness seemed to him to be thus involved, and with them the *principia individuationis*, space and time. Now in both premise and inference Kant may have been quite wrong; but he put a question that had depth and dignity, he suggested a method of discussing it, and the philosopher with historical training finds it difficult to repress a certain impatience when he is asked to compare Kant's results with those obtained by a method which may indeed employ certain terms in common with Kant, but which has evidently been conceived with no comprehension of what this deep but crabbed old thinker was struggling to express.

If I have referred frequently to the contrast between Mach's empiricism and the results of Kant, it is because Mach himself has chosen this means of orienting himself historically. But Kant is an accident: had he not invented a certain way of putting the question, the mere evolution of the type of empiricism we are here dealing with would have forced *us* to invent one. In a characteristic chapter Mach pictures the growth of knowledge as an "Anpassung der Gedanken an die Tatsachen und aneinander" (p. 162 ff.). The phrase serves admirably to convey a truth respecting the evolution of science, and to call attention to an analogy between widely differing examples of adaptation. The scientific interest of this analogy can be appreciated by one who has never stopped to enquire what this term 'fact' means, to which our science and our organism equally adapt themselves. It is sufficient to assume that whatever 'fact' means in the one case it means in the other. But there are connections in which we are not content with establishing a sameness of meaning, but in which we demand the common definition of 'fact.' When the demand is brought to bear on any illustration the empiricist may offer of an "Anpassung der Gedanken an die Tatsachen," we seem to discover that the situation would serve equally well to illustrate an "Anpassing der Tatsachen an die Gedanken." Whither does this discovery lead us? It may not lead us to Kant, certainly,— he worked more than a century ago. But that Kant did his part in leading us to the discovery is one of his chief claims on our gratitude. Neither the general question nor Kant's peculiar contribution to it can be discussed on the basis of an empiricism that has never stopped to ask itself, 'What is a fact?'

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La moralisme de Kant et l'amoralisme contemporain. Par ALFRED FOUILLÉE. Paris, Félix Alcan, 1905. — pp. xxiii, 375.

According to M. Fouillée, contemporary ethical theories may all be classed under one of two heads, the moralism of Kant and his fol-